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Theoretical and experimental constitutive equations of superplastic behaviour: discussion

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Two basic models of superplastic deformation are considered. The stress strain-rate curves obtained by combining these two models for a material of different grain sizes at various temperatures are shown to be very different. This is due to the introduction of physically ill-defined parameters in the theoretical relations. The experimental results obtained from mechanical tests do not help in clarifying the situation since the microstructure is continually changing during such experiments. Consequently, the validity of the deformation mechanisms in dealing with models of constant grain-size cannot be demonstrated by fitting theoretical curves to the experimental results of mechanical tests.

1. Introduction

All the experimental stress strain-rate data for superplastic materials show, in general, three distinct regimes as shown in Fig. 1 [1]. In the third regime, their behaviour is similar to conventional materials deformed at high temperature. The second regime is characterized by high values of the strain-rate sensitivity parameter whereas in the first regime, the deformation occurs at an extremely low strain-rate. Therefore, very few investigations have been carried out to study the deformation mechanism in this case.

Different models have been proposed to explain the behaviour of superplastic materials; they led to a relation between stress, strain-rate, temperature and grain size of the material. Each of these models usually applies to one of the three regimes shown in Fig. 1. Some investigators have tried to explain the stress strain-rate behaviour over the three regimes by combining these different models.

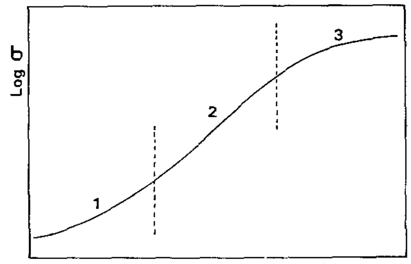

Figure 1 Schematic variation of stress versus strain-rate.

The aim of the present work is to discuss the validity of two theoretical models [2, 3] which have been proposed to explain superplastic deformation in the light of microstructural considerations.

2. Theoretical variations of stress with strain-rate

Only the most recent models formulated to explain the deformation in the second and third regime of superplastic behaviour will be considered. In the third regime, different dislocation creep models have been employed to explain this behaviour. A relation proposed by Weertman [4]

$$\dot{\epsilon}_{DC} = \alpha D_V \left(\frac{\sigma}{\mu}\right)^{4.5} \frac{\mu \Omega}{kT} \quad (1)$$

where α is a constant, μ is the shear modulus, D_V the lattice self-diffusion coefficient, Ω the atomic volume, k Boltzman's constant and T the absolute temperature, has been widely used.

In the second regime, all experimental observations show that grain-boundary sliding plays an important role during deformation. To prevent formation of cavities at triple junctions, grain-boundary sliding may be accompanied by controlling mechanisms such as boundary migration, lattice or boundary diffusion, dislocation creep in neighbouring grains. Recently, two models have been proposed: a grain-boundary sliding model accompanied by diffusion (G.S.D.) suggested by Ashby and Verrall [2] and a grain-boundary sliding model accompanied by dislocation creep in neighbouring grains (G.S.D.C.) suggested by Hayden *et al.* [3]. Mathematical analysis of these models have led to the following relations:

$$\dot{\epsilon}_{\mathrm{GSD}} = \frac{100\Omega}{kTd^2}(\sigma - \sigma_0)D_{\mathrm{V}} \left(1 + \frac{3.3\delta}{d}\frac{D_{\mathrm{B}}}{D_{\mathrm{V}}}\right) \quad (2)$$

$$\dot{\epsilon}_{\mathrm{GSDC_1}} = \left(\frac{K_1}{d} + 1\right) \frac{24\pi(1-\nu)D_{\mathrm{B}}\,\boldsymbol{b}^3\,\sigma(\sigma - \sigma'_0)}{d^2\mu kT} \text{ for } T < T_{\mathrm{c}} \quad (3)$$

$$\dot{\epsilon}_{\mathrm{GSDC_2}} = \left(\frac{K_1}{d} + 1\right) \frac{6\pi(1-\nu)D_{\mathrm{V}}\,\boldsymbol{b}^2\,\sigma(\sigma - \sigma'_0)}{10d\mu kT} \text{ for } T > T_{\mathrm{c}} \quad (4)$$

where d is the grain size, ν Poisson's ratio, $\boldsymbol{b}$ Burgers vector and δ the thickness of the boundary as a high diffusivity path. σ_0 and σ'_0 are threshold stresses below which no plastic flow takes place. In the above relations, T_{c} is a transition temperature above which the lattice diffusion prevails.

These different relations for the second and third regime (Equations 1 to 4) describe independent flow mechanisms which can occur simultaneously. In such case, the overall strain-rate is given by:

$$\dot{\epsilon}_{\mathrm{T}} = \dot{\epsilon}_{\mathrm{GSD}} + \dot{\epsilon}_{\mathrm{GSDC}} + \dot{\epsilon}_{\mathrm{D.C}}\,. \quad (5)$$

These relations employ different parameters which characterize the material: K_1, δ, D_{V}, D_{B}, σ_0 and σ'_0. Each parameter is discussed separately below.

K_1, a coefficient which has length dimension is a measure of the proportionality between strain-rate due to grain-boundary sliding and strain-rate due to dislocation creep. The value of K_1 is not very well defined and the authors [3] estimate its value at 150 μm for various superplastic materials.

δ is defined as the thickness of the boundary as a high diffusivity path. The boundary theories are still very much under discussion and it appears very difficult to give a value to this parameter. A value of $2\boldsymbol{b}$ may be a reasonable value as indicated by Ashby and Verrall [2].

The self-diffusion coefficients D_{V} and D_{B} are in the form

$$D_{\mathrm{V}} = D_{0\mathrm{V}} \exp\left(-\frac{Q_{\mathrm{V}}}{kT}\right) \quad (6)$$

$$D_{\mathrm{B}} = D_{0\mathrm{B}} \exp\left(-\frac{Q_{\mathrm{B}}}{kT}\right). \quad (7)$$

The experimental values of $D_{0\mathrm{V}}$ and Q_{V} are relatively well-established for single-phased alloys. On the other hand, $D_{0\mathrm{B}}$ and Q_{B} are difficult to determine. However, it is generally admitted that Q_{B} is equal to $Q_{\mathrm{V}}/2$. Subsequently, we may give to $Q_{0\mathrm{V}}$, Q_{V}, $D_{0\mathrm{B}}$ and Q_{B} the following values respectively [3]:
1 cm^2 sec^{-1}, 1 eV, 2×10^{-2} cm^2 sec^{-1}, 0.5 eV

σ_0 the threshold stress which appears in Ashby and Verrall's model [2] is given by $\sigma_0 = 0.72\Gamma/d$ where Γ is the boundary free energy. This term will be due to boundary area variation during deformation. According to the authors, it seems necessary to take into account the fact that the boundary is not a perfect source (or sink) for vacancies. The boundary structure is not known and it must be quite difficult to give a value to σ_0. In Hayden *et al.*'s model, σ'_0 is either the stress necessary to nucleate a dislocation from a grain-boundary source or the lattice friction stress [3]. Burton attempted to determine a back-stress for the eutectic alloy PbSn by creep tests under very small stresses [5]. The back-stress was determined at very small strains and strain-rates (less than 10^{-9} sec^{-1}). Therefore, the value determined is not very reliable since at a strain less than 1%, the material obviously does not reach the steady state [6-8]. Subsequently, we consider σ_0 and σ'_0 as parameters.

Figs. 2 and 3 show the influence of temperature and grain-size on the stress-strain rate curves respectively, when σ_0 and σ'_0 may be considered as negligible with respect to the applied stress. These curves show that at a given stress, the strain-rate in the superplastic range (regime II) is higher for finer grain size or higher temperature. It should be noted that changing the numerical values of K_1, δ, D_{V} and D_{B} will shift the curves either with respect to each other or to the ordinate, but without changing their overall shape.

The shape of the previous curves will change

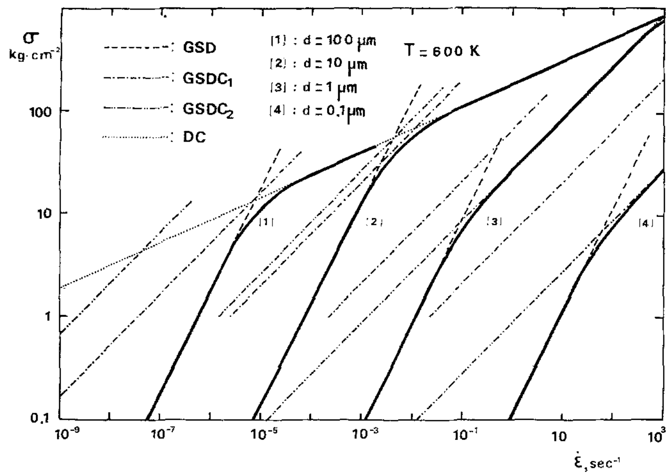

Figure 2 Theoretical variations of stress versus strain-rate for materials of different grain sizes; σ_0 and σ'_0 are negligible.

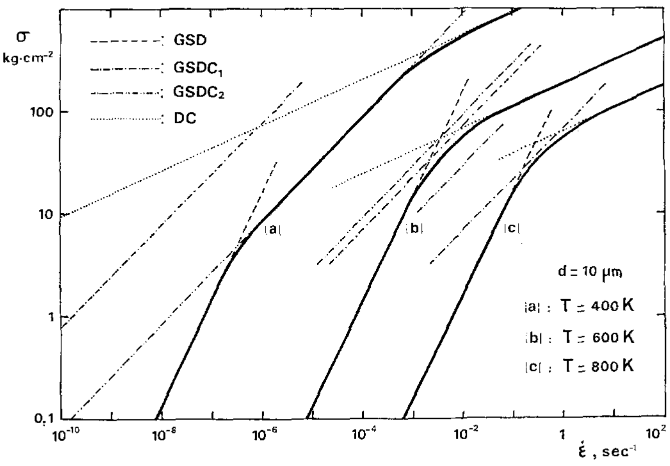

Figure 3 Theoretical variations of stress versus strain-rate at different temperatures; σ_0 and σ'_0 are negligible.

when σ_0 and/or σ'_0 are not negligible with respect to the applied stress. To illustrate this, two curves are represented in Figs. 4 and 5. In Fig. 4, σ_0 is greater than σ'_0, and in Fig. 5, the opposite case is considered. These stress strain-rate curves exhibit three regimes. At high stresses, the dislocation creep is the predominant deformation mechanism. At intermediate stress levels, two cases may occur according to the values attributed to σ_0 and σ'_0:

$\dot{\epsilon} \propto \sigma^2$ which corresponds to $m = \frac{1}{2}$

$\dot{\epsilon} \propto \sigma$ which corresponds to $m = 1$.

At low flow-stress levels, a deformation regime I (Fig. 1) with low strain-rate sensitivity parameters takes place. This deformation regime may, in fact, be a continuation of regime II, and arising only from the existence of the threshold stress below which no flow is possible. This idea has been previously reported [9, 10].

For certain values of σ_0 and σ'_0, we may find a more complicated stress strain-rate curve as shown in Fig. 6. This curve is then divided into five regimes, two of them being characterized by high values of the strain-rate sensitivity index.

This study shows the complexity and the diversity of stress-strain rate curves which may be obtained from the proposed theoretical models. Curves of the same general form could

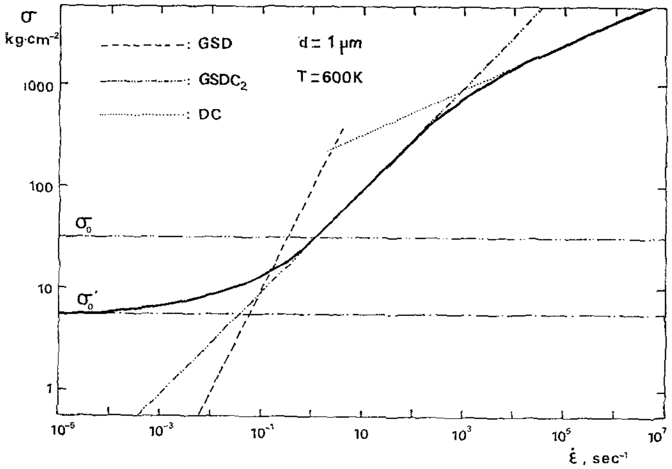

Figure 4 Theoretical variations of stress versus strain-rate $\sigma_0 > \sigma'_0$.

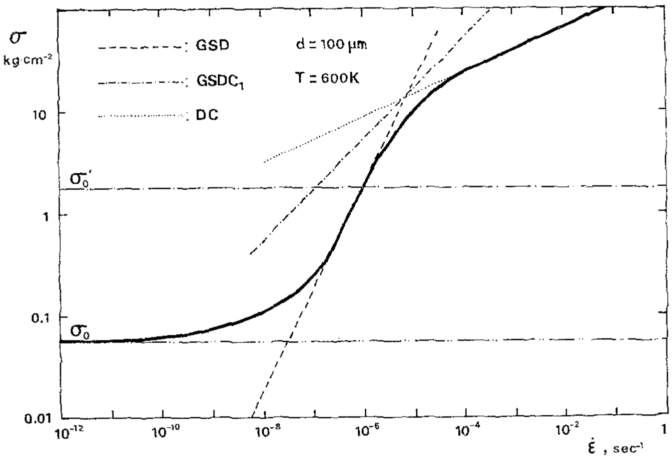

Figure 5 Theoretical variations of stress versus strain-rate $\sigma_0 < \sigma'_0$.

also be obtained from basic models other than the two considered above.

The verification of a proposed theoretical model is usually established by comparison with experimental results. We have discussed above two models suggested to explain the superplastic deformation. In the following section, we will discuss the various methods by which these experimental results are usually obtained.

3. Experimental variations of stress with strain-rate

The stress strain-rate curves of a superplastic material of a given initial grain size are usually obtained by tensile tests either at constant cross-head speed or constant strain-rate. The method consists of plotting the flow stress at an arbitrarily chosen strain versus the strain-rate. The strain-rate sensitivity index is also determined either from differentiating the stress strain-rate curves or by the velocity step-change method which involves controversial extrapolations [11].

All experimental stress strain-rate curves of superplastic behaviour show that:

(1) both the flow stress and strain-rate sensitivity parameter are strain-dependent. This observation is not considered in formulating the previous theoretical models;

(2) the values of the strain-rate sensitivity

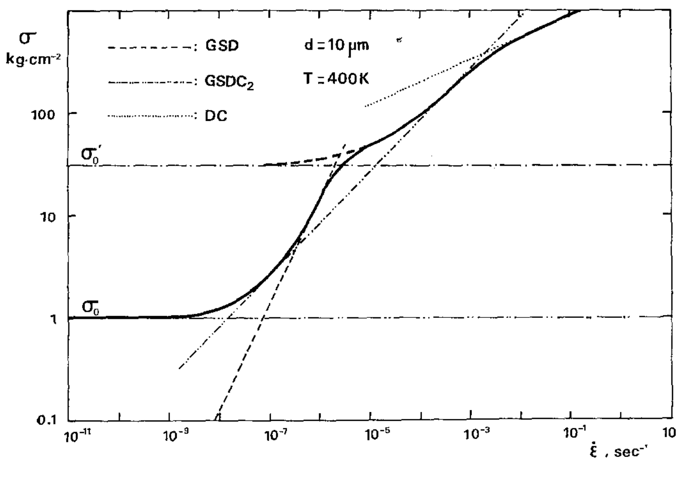

Figure 6 Theoretical variations of stress versus strain-rate $\sigma_0 < \sigma'_0$.

parameter depend on the method of determination [12]. This dependence has been demonstrated by Dunlop and Taplin [12] and explained in terms of non-uniform elongation;

(3) the experimental values of the strain-rate sensitivity index vary continuously with respect to strain-rate; this observation seems to indicate that no deformation mechanism would be predominant in any strain-rate range.

4. Discussion

The above three observations present the most important contradictions to any theoretical model suggested to explain the deformation mechanisms of superplastic behaviour. However, it seems to us that the discrepancy between theory and experiments is due to the oversimplified theoretical models which usually apply only to a single-phase, equiaxed and constant grain size material. However, most superplastic materials are generally two-phased and it has been recently shown that the phases may play different roles during superplastic deformation [6, 13-15]. In the isolated cases of superplastic materials having a predominant phase such as Sn 1% Bi and which may be considered as single-phased materials, the structure is not stable at all [16-19]. Structural changes, specifically grain-coarsening, usually take place even in the case of two-phased materials during superplastic deformation [6, 20-26]. This coarsening is a decreasing function of strain-rate and results in an increase of flow stress [8, 16]. Recent studies have shown that some materials obtained by extrusion or rolling, exhibit structure with elongated phases. After superplastic tensile deformation of about 20% elongation, the structure becomes approximately equiaxed. This process is accompanied by a decrease in flow stress [8]. A theoretical analysis of this shape change has been proposed [3].

These structural changes during superplastic deformation show that it is impossible to characterize the material by a stable grain size. Hence, the constitutive law should necessarily take into account these structural changes and, therefore, the flow stress at constant temperature may be given in the form:

$$\sigma = \sigma(\dot{\epsilon}, S)_T \,. \tag{8}$$

On a logarithmic diagram, an increment of flow-stress $d\log\sigma$ may be considered as the sum of two increments: one $d\log\sigma_r$ due to the fact that the material is strain-rate sensitive and the other $d\log\sigma_S$ due to structural changes, i.e.

$$d\log\sigma = d\log\sigma_r + d\log\sigma_S \,. \tag{9}$$

Assuming that a power-law, $\sigma_r = K\dot{\epsilon}^m$ may represent the stress strain-rate behaviour of a superplastic material of constant microstructure, then:

$$d\log\sigma = m\,d\log\dot{\epsilon} + d\log\sigma_S \,. \tag{10}$$

The stress increment $d\log\sigma_S$ depends on strain and strain-rate or, more generally, on deformation path. The strain-rate sensitivity index is defined as:

$$m = \left(\frac{\partial\log\sigma}{\partial\log\dot{\epsilon}}\right)_{S,\,T} \,. \tag{11}$$

The conventional methods used to measure the strain-rate sensitivity parameter lead to values which are characteristic of the structure as measurement is being made. Therefore, these values cannot be compared as they correspond to different structures of the material. Generally, they are only associated with the same initial structure S_i. The strain-rate sensitivity parameter is then given by:

$$m' = \left(\frac{\partial \log\sigma}{\partial \log\dot{\epsilon}}\right)_{S_i,\,T}. \tag{12}$$

This value obviously differs from that given by Equation 11. They become identical only in the case where corrections for the microstructural changes are considered.

A recent study [27] has shown that by taking into account the structural changes of the different phases, constant values of the strain-rate sensitivity parameter, m, as defined by Equation 11 may be obtained over a very wide range of strain-rate. This study was carried out on the CuP alloy which shows two phases in the considered range, one Cu_3P is intermetallic, the other, the α-phase, is a solid solution of phosphorus in copper. This alloy seems very interesting because the phases present very different ductilities.

In order to have a very wide range of phase sizes, three phosphorus concentrations about the eutectic composition (8.4 wt % P) have been chosen and different thermomechanical processes have been applied. These treatments are shown in Fig. 7.

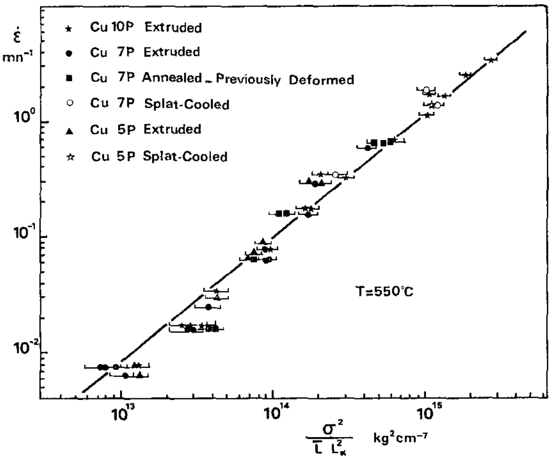

Figure 7 Relation between mechanical and structural parameters of CuP alloy of different compositions and after different thermomechanical treatments, deformed in the superplastic range.

In the superplastic range, the strain-rate $\dot{\epsilon}$ is linearly correlated with the expression $\sigma^2/(\bar{L}L_\alpha^2)$. In this expression, σ is the flow stress, $\bar{L}$ and $\bar{L}_\alpha$ are parameters which characterize the structure: L_α is the average size of the α phase and $\bar{L}$ is the mean distance between a Cu_3P grain and the neighbouring grains (α or Cu_3P). The choice of these parameters to describe the structure will be justified in a future article [27]. The mechanical parameters, σ and $\dot{\epsilon}$, have been determined during tensile tests at constant strain-rate before quenching the specimen. $\bar{L}$ and L_α have been determined after quenching on cuts that are normal and parallel to tensile axis.

This curve shows that in the superplastic range there is a single relation between the mechanical and structural parameters over a wide range of strain-rate when the evolution of the structure during the tensile tests is carefully taken into account. In particular, the strain-rate sensitivity coefficient, m, is equal to 0.5. Later theoretical considerations will justify this relation.

5. Conclusions

By considering two models of superplastic deformation, it is shown theoretically that a large variety of shapes of stress strain-rate curves could be produced. This is mainly due to the introduction of physically ill-defined parameters in the models.

The theoretical models are only valid in the case of a material which deforms with constant microstructure. Therefore, successful fitting of such a model and the experimental results should certainly be viewed as controversial.

A rigorous theoretical model should introduce parameters which compensate for microstructural changes during deformation. These parameters must be obviously amenable to evaluation from a sequence of microscopic observations.

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